Lesson 8-3 Parts 1 & 2 Learning Check

AP Calculus AB
Lesson 8-3, Parts 1 & 2 Learning Check

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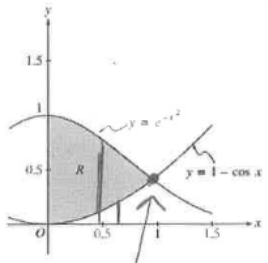
Date

For now, skip problems (1c), (2c), and (3b) – we will come back to them later. Attempt all remaining parts!

CALCULATOR ACTIVE

1. Let R be the shaded region in the first quadrant enclosed by the graphs of $y = e^{-x^2}$, $y = 1 - \cos x$, and the y-axis, as shown in the figure above.
   (a) Find the area of the region R.
   (b) Find the volume of the solid generated when the region R is revolved about the x-axis.
   (c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a square. Find the volume of this solid.



a. $A = \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$

$= .590$ or $.591$

use calculator to get intersection. Store : $X = .941944$ as A

$e^{-x^2} = 1 - \cos x$

b. $V = \pi \int_0^A \left((e^{-x^2})^2 - (1 - \cos x)^2\right) dx$

$= .55596\pi$

≈ 1.746 or 1.747

★ right click on x value of intersection
– #5 ⇒ Store
– choose variable
A
– Press Enter

OVER →

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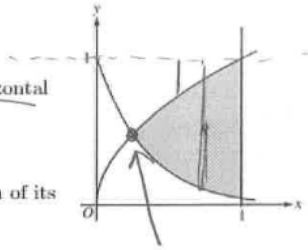
CALCULATOR ACTIVE

2. Let R be the shaded region bounded by the graphs of $y = \sqrt{x}$ and $y = e^{-3x}$ and the vertical line $x = 1$, as shown in the figure above.



(a) Find the area of R.

(b) Find the volume of the solid generated when R is revolved about the horizontal line $y = 1$.

(c) The region R is the base of a solid. For this solid, each cross section perpendicular to the x-axis is a rectangle whose height is 5 times the length of its base in region R. Find the volume of this solid.

a. $A = \int_A^1 (\sqrt{x} - e^{-3x})\,dx$

$\approx .442$ or $.443$

(0.238737, .488604)

Store as A

$\sqrt{x} = e^{-3x}$

b. $V = \pi \int_A^1 ((1 - e^{-3x})^2 - (1 - \sqrt{x})^2)\,dx$

$= .453\pi$

≈ 1.423 or 1.424

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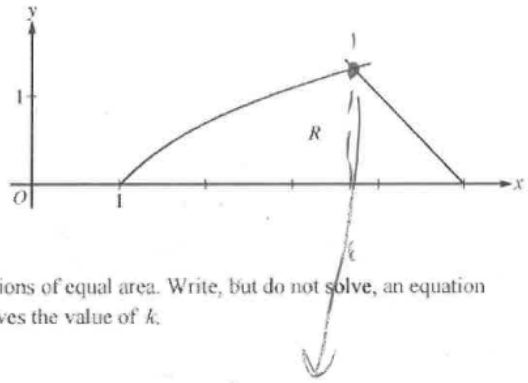
CALCULATOR ACTIVE

3. Let R be the region in the first quadrant bounded by the x-axis and the graphs of $y = \ln x$ and $y = 5 - x$, as shown in the figure above.



(a) Find the area of R.

(b) Region R is the base of a solid. For the solid, each cross section perpendicular to the x-axis is a square. Write, but do not evaluate, an expression involving one or more integrals that gives the volume of the solid.

(c) The horizontal line $y = k$ divides R into two regions of equal area. Write, but do not solve, an equation involving one or more integrals whose solution gives the value of k.

$\ln x = 5 - x$

$x \approx 3.69344$

Store as A

a. Integrate w.r.t. y.

$y = 5 - x$ $\quad y = \ln x$

$x = 5 - y$ $\quad x = e^y$

$$A = \int_0^A (5 - y - e^y)\,dy$$

≈ 2.986 or 2.985

or

$$A = \int_1^A \ln x\,dx + \int_A^5 (5 - x)\,dx$$

c. $$\int_0^K (5 - y - e^y)\,dy = \frac{1}{2} \cdot 2.986$$

Lesson 8-3 Parts 1 & 2 Learning Check

AP Calculus AB
8-3 *Volumes of Revolution* Learning Check ANSWERS

1.

Region R $e^{-x^2} = 1 - \cos x$ at $x = 0.941944 = A$	1 : Correct limits in an integral in (a), (b), or (c).
(a) Area $= \int_0^A (e^{-x^2} - (1 - \cos x))\,dx$ $= 0.590$ or 0.591	2 { 1 : integrand 1 : answer
(b) Volume $= \pi \int_0^A \left(\left(e^{-x^2}\right)^2 - (1 - \cos x)^2\right) dx$ $= 0.55596\pi = 1.746$ or 1.747	3 { 2 : integrand and constant $< -1 >$ each error 1 : answer
(c) Volume $= \int_0^A \left(e^{-x^2} - (1 - \cos x)\right)^2 dx$ $= 0.461$	3 { 2 : integrand $< -1 >$ each error Note: 0/2 if not of the form $k\int_c^d (f(x) - g(x))^2\,dx$ 1 : answer

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2. Point of intersection
$e^{-3x} = \sqrt{x}$ at $(T, S) = (0.238734, 0.488604)$

(a) Area $= \int_T^1 \left(\sqrt{x} - e^{-3x}\right) dx$
$= 0.442$ or 0.443

(b) Volume $= \pi \int_T^1 \left(\left(1 - e^{-3x}\right)^2 - \left(1 - \sqrt{x}\right)^2\right) dx$
$= 0.453\pi$ or 1.423 or 1.424

(c) Length $= \sqrt{x} - e^{-3x}$
Height $= 5\left(\sqrt{x} - e^{-3x}\right)$

Volume $= \int_T^1 5\left(\sqrt{x} - e^{-3x}\right)^2 dx = 1.554$

1: Correct limits in an integral in (a), (b), or (c)

2 : { 1 : integrand; 1 : answer

3 : { 2 : integrand; $< -1 >$ reversal; $< -1 >$ error with constant; $< -1 >$ omits 1 in one radius; $< -2 >$ other errors; 1 : answer

3 : { 2 : integrand; $< -1 >$ incorrect but has $\sqrt{x} - e^{-3x}$ as a factor; 1 : answer

Lesson 8-3 Parts 1 & 2 Learning Check

3.

$\ln x = 5 - x \Rightarrow x = 3.69344$

Therefore, the graphs of $y = \ln x$ and $y = 5 - x$ intersect in the first quadrant at the point $(A, B) = (3.69344, 1.30656)$.

(a) Area $= \int_0^B (5 - y - e^y)\, dy$
$= 2.986$ (or 2.985)

OR

Area $= \int_1^A \ln x\, dx + \int_A^5 (5 - x)\, dx$
$= 2.986$ (or 2.985)

3 : { 1 : integrand; 1 : limits; 1 : answer }

(b) Volume $= \int_1^A (\ln x)^2\, dx + \int_A^5 (5 - x)^2\, dx$

3 : { 2 : integrands; 1 : expression for total volume }

(c) $\int_0^k (5 - y - e^y)\, dy = \frac{1}{2} \cdot 2.986 \left(\text{or } \frac{1}{2} \cdot 2.985\right)$

3 : { 1 : integrand; 1 : limits; 1 : equation }